

# Mathematical modeling: a very quick intro

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# I am not a modeler, but have been part of modeling studies

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PLOS MEDICINE

## The Importance of Implementation Strategy in Scaling Up Xpert MTB/RIF for Diagnosis of Tuberculosis in the Indian Health-Care System: A Transmission Model

Henrik Salje<sup>1</sup>, Jason R. Andrews<sup>2\*</sup>, Sarang Deo<sup>3</sup>, Srinath Satyanarayana<sup>4,5</sup>, Amanda Y. Sun<sup>6</sup>, Madhukar Pai<sup>4,5,7\*</sup>, David W. Dowdy<sup>1,8\*</sup>

PLOS ONE

RESEARCH ARTICLE

## Xpert<sup>®</sup> MTB/RIF for the Diagnosis of Tuberculosis in a Remote Arctic Setting: Impact on Cost and Time to Treatment Initiation

Olivia Oxlade<sup>1,2</sup>, Jordan Sugarman<sup>1</sup>, Gonzalo G. Alvarez<sup>3,4</sup>, Madhukar Pai<sup>1,2</sup>, Kevin Schwartzman<sup>1,2\*</sup>

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PLOS MEDICINE

## Serological Testing Versus Other Strategies for Diagnosis of Active Tuberculosis in India: A Cost-Effectiveness Analysis

David W. Dowdy<sup>1</sup>, Karen R. Steingart<sup>2</sup>, Madhukar Pai<sup>3\*</sup>



American Journal of Epidemiology  
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Vol. 178, No. 12  
DOI: 10.1093/aje/kwt210  
Advance Access publication:  
October 7, 2013

Practice of Epidemiology

## Modeling the Impact of Alternative Strategies for Rapid Molecular Diagnosis of Tuberculosis in Southeast Asia

Amanda Y. Sun, Madhukar Pai, Henrik Salje, Srinath Satyanarayana, Sarang Deo, and David W. Dowdy\*

# The potential impact of health service disruptions on the burden of malaria:

a modelling analysis for countries in sub-Saharan Africa



Developed by Stop TB Partnership in collaboration with Imperial College, Avenir Health, Johns Hopkins University and USAID.

**THE POTENTIAL IMPACT OF THE COVID-19 RESPONSE ON TUBERCULOSIS IN HIGH-BURDEN COUNTRIES: A MODELLING ANALYSIS**

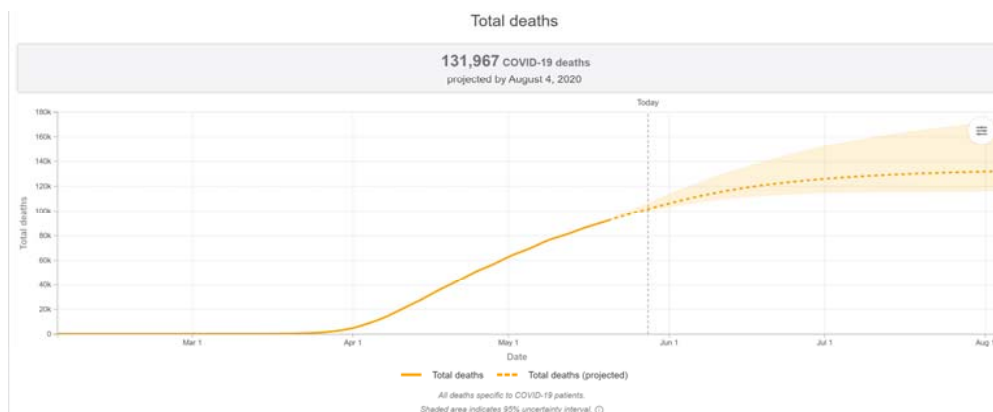
16 March 2020

Imperial College COVID-19 Response Team

## Report 9: Impact of non-pharmaceutical interventions (NPIs) to reduce COVID-19 mortality and healthcare demand

Neil M Ferguson, Daniel Laydon, Gemma Nedjati-Gilani, Natsuko Imai, Kylie Ainslie, Marc Baguelin, Sangeeta Bhatia, Adhiratha Boonyasiri, Zulma Cucunubá, Gina Cuomo-Dannenburg, Amy Dighe, Ilaria Dorigatti, Han Fu, Katy Gaythorpe, Will Green, Arran Hamlet, Wes Hinsley, Lucy C Okell, Sabine van Elsland, Hayley Thompson, Robert Verity, Erik Volz, Haowei Wang, Yuanrong Wang, Patrick GT Walker, Caroline Walters, Peter Winskill, Charles Whittaker, Christl A Donnelly, Steven Riley, Azra C Ghani.

On behalf of the Imperial College COVID-19 Response Team



# Flood of Covid-19 modeling analyses

# Models as a scaffold or thinking framework

- Models are **not** designed to predict the future, nor to represent the full complexity of the world.
  - Simpler is often better, as only models that are sufficiently simple can be useful to the average decision-maker.
- Rather, models provide a decision-making framework that is:
  - Systematic
  - Data-driven
  - Transparent
  - Comprehensible
  - Reproducible
  - Able to describe uncertainty and key data needs



# A Modeling Exercise

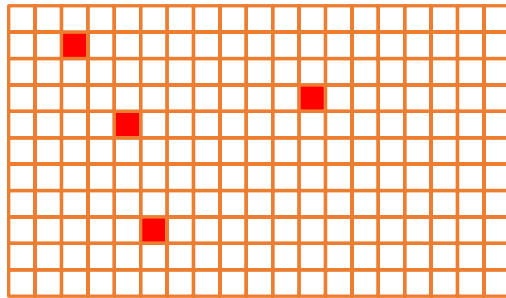
- Consider two diagnostic tests for TB:
  - Test 1: 50% sensitive, but can be used to screen the population once per year (in addition to self-presentation)
  - Test 2: 90% sensitive, but requires patients to self-present
- A few basic assumptions:
  - Duration of TB disease before self-presentation = 1 year
  - Number of secondary transmissions per infectious person-year = 10
  - Proportion of secondary transmissions that result in active TB = 20%
    - 1 new case of active TB every 6 months
  - Mortality of TB without screening at the end of year 1:
    - 20% if detected
    - 100% if not detected
- Which test is better?

# The Simple Approach...

- Evaluate a cohort of 100 people

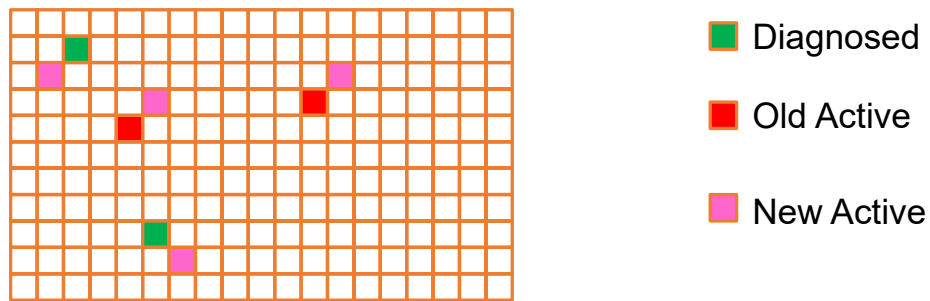
<b>Outcome</b>	<b>Test 1</b>	<b>Test 2</b>
Screened	50	0
Diagnosed after 1 year	25	90
Survive	20	72
Die	5	18
Undiagnosed = dead	25	10
<b>Total Alive</b>	<b>70</b>	<b>72</b>
<b>Total Dead</b>	<b>30</b>	<b>28</b>

# What If We're Interested in the Population?



**4 TB cases in a community with test 1**

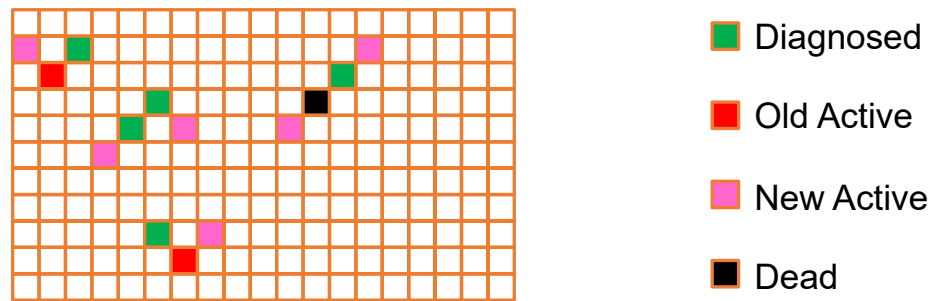
# Population Dynamics



**At 6 Months: 6 Active Cases**

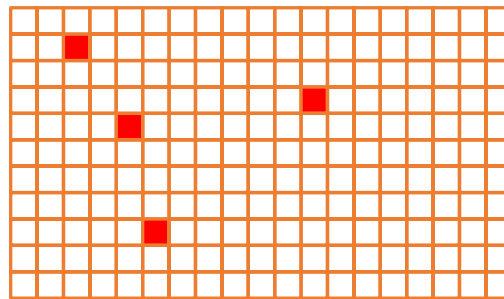


# Population Dynamics



**At 12 Months: 8 Active Cases**

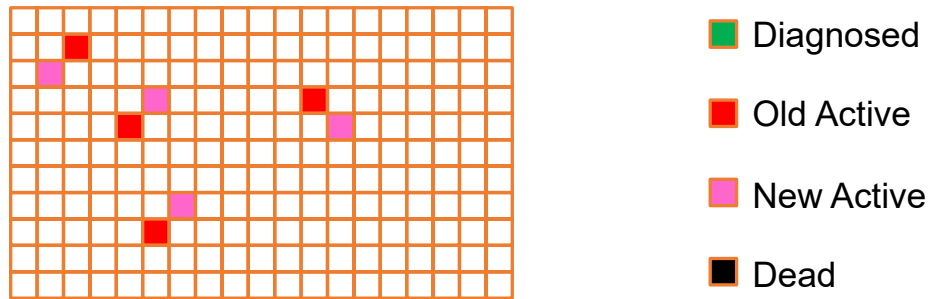
# Alternative Population Dynamics



- Diagnosed
- Old Active
- New Active
- Dead

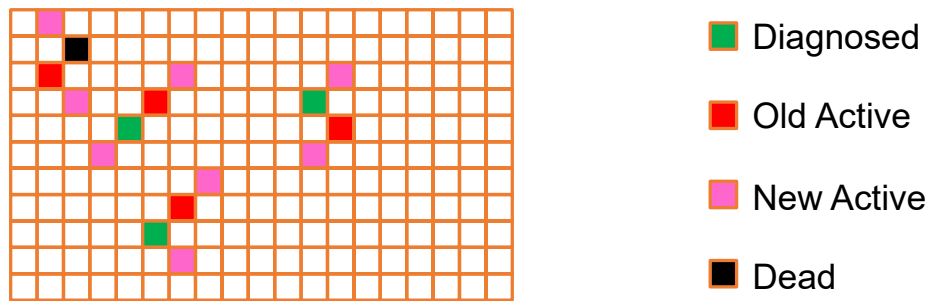
**What about test 2?**

# Alternative Population Dynamics



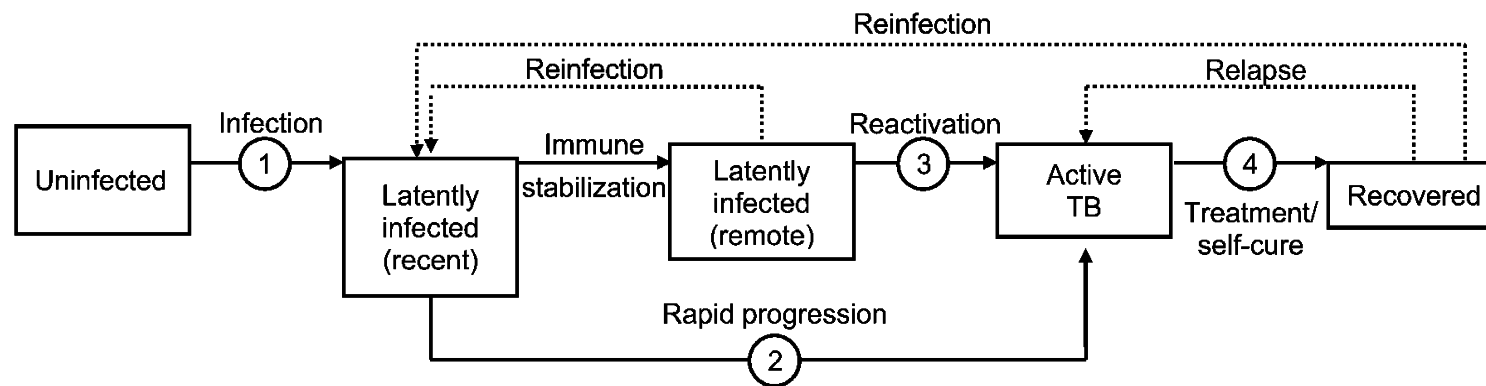
**6 Months: 8 Active Cases**

# Alternative Population Dynamics



**12 Months: 12 Active Cases!**

# This is a compartmental or SIR model SIR (Susceptible, Infectious, or Recovered)

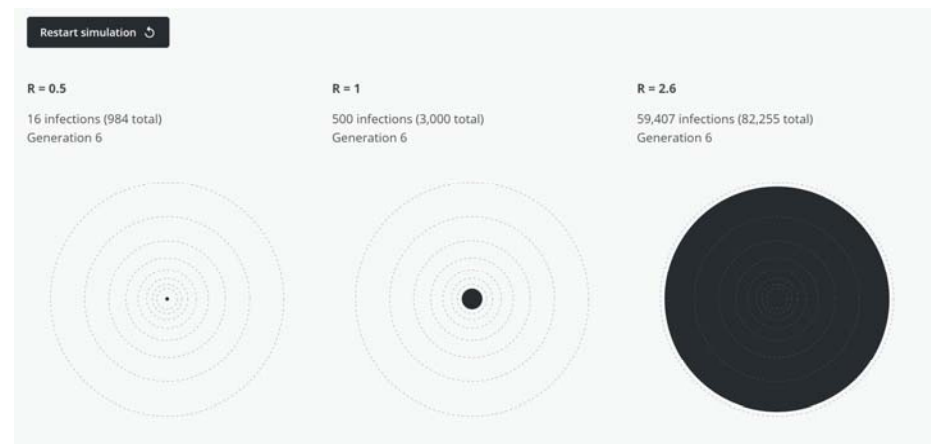


- Re-calculate the population every few days
- As the size of the active TB compartment grows, the rate of infection grows
- Allows for real-time evaluation of population-level effects on transmission

# R, a key metric to watch as COVID-19 restrictions are lifted

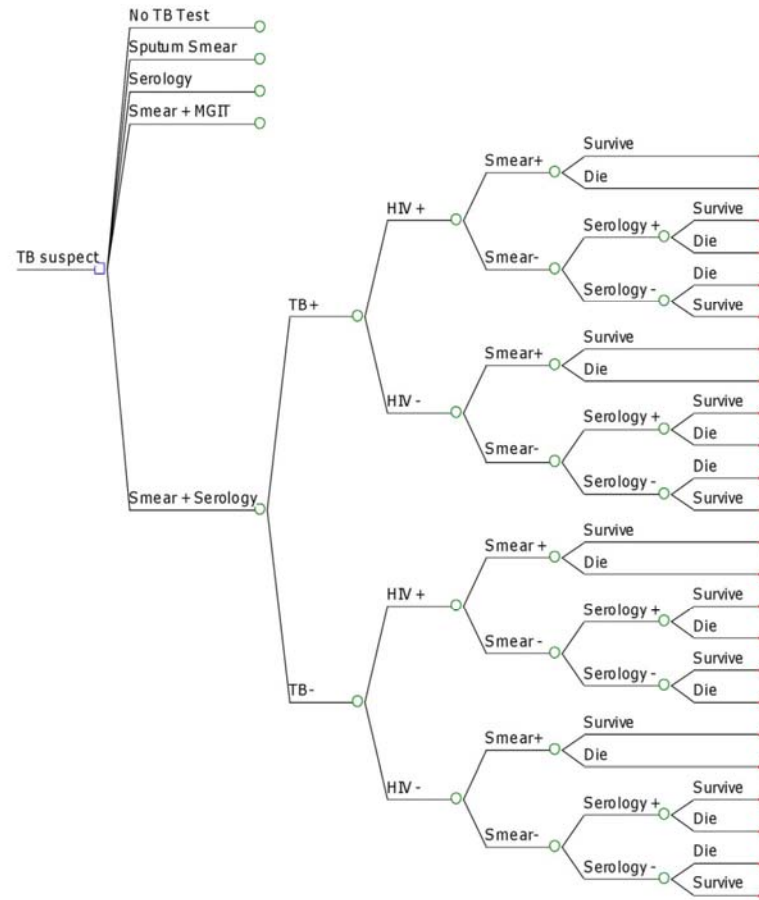
Reproduction number can help decision-makers know when it's safe to loosen restrictions and when it's not

Emily Chung  
CBC News • May 27, 2020



<https://newsinteractives.cbc.ca/coronavirus0/>

# This is a decision analysis model



# All good models must

- Describe the setting & population
- Limit to a certain time horizon
- Describe and quantify all the assumptions and justify them
- Provide projections with uncertainty ranges
- Do sensitivity analyses by varying the assumptions
- Discuss limitations of the model
- Publish the code and data used for others to verify



The NEW ENGLAND JOURNAL of MEDICINE

## Perspective

### Wrong but Useful — What Covid-19 Epidemiologic Models Can and Cannot Tell Us

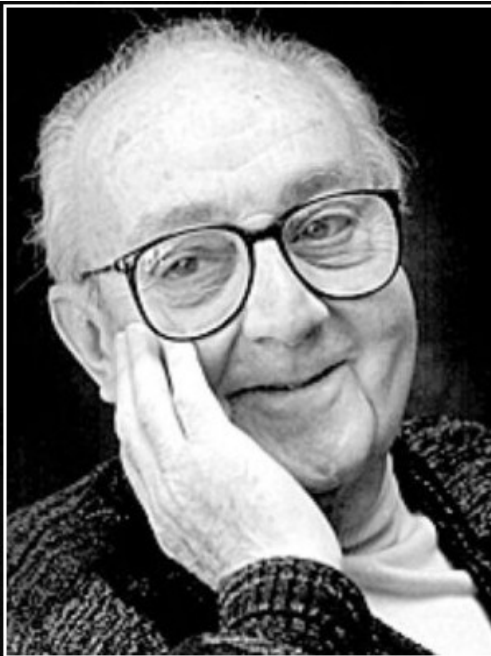
Inga Holmdahl, S.M., and Caroline Buckee, D.Phil.

#### Five Questions to Ask about Model Results.

1. What is the purpose and time frame of this model? For example, is it a purely statistical model intended to provide short-term forecasts or a mechanistic model investigating future scenarios? These two types of models have different limitations.
2. What are the basic model assumptions? What is being assumed about immunity and asymptomatic transmission, for example? How are contact parameters included?
3. How is uncertainty being displayed? For statistical models, how are confidence intervals calculated and displayed? Uncertainty should increase as we move into the future. For mechanistic models, what parameters are being varied? Reliable modeling descriptions will usually include a table of parameter ranges — check to see whether those ranges make sense.
4. If the model is fitted to data, which data are used? Models fitted to confirmed Covid-19 cases are unlikely to be reliable. Models fitted to hospitalization or death data may be more reliable, but their reliability will depend on the setting.
5. Is the model general, or does it reflect a particular context? If the latter, is the spatial scale — national, regional, or local — appropriate for the modeling questions being asked and are the assumptions relevant for the setting? Population density will play an important role in determining model appropriateness, for example, and contact-rate parameters are likely to be context-specific.

# What makes Covid-19 modeling challenging?

- New disease – everyone is learning
- $R_0$  is not static
- Testing rates are changing
- Natural protective immunity: unclear
- % of population infected: unclear
- Case fatality: highly variable
- Do asymptomatic people infect others?
- Duration of infectiousness?
- Do children transmit infection?
- How many people come into contact with a case?
- Are there 'super-spreaders'?
- Lockdowns, restrictions & other interventions keep changing
- Human behaviour!!!!



All models are wrong, but some are  
useful.

— *George E. P. Box* —

AZ QUOTES

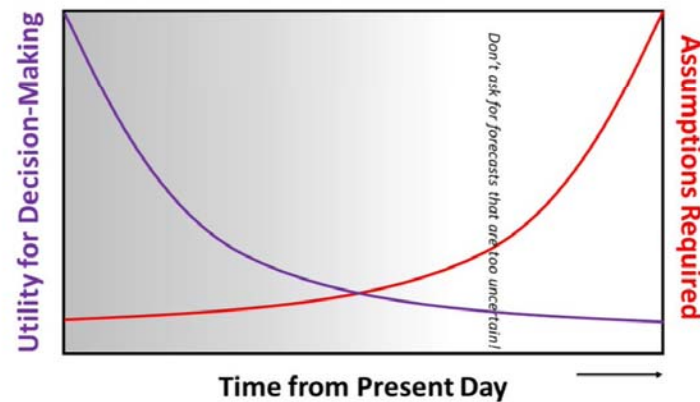
# We should not have crazy expectations!



**David Dowdy** @davidwdowdy · May 3

We've been collecting detailed weather data for decades, yet we don't expect weather forecasters to predict the weather more than 1 week in advance. Why do we ask COVID epidemic modelers to forecast case counts many months from now, when we have far fewer data?

## *Diminishing Utility of Forecasts with Increasing Time*



6

20

